Paraconsistent Logics for Knowledge Representation and Reasoning:

advances and perspectives

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Introduction

Non-classical Logics for KRR

» Non-classical logics find several applications in artificial intelligence, including multi-agent systems, reasoning with vagueness, uncertainty, and contradictions, among others.

» Regarding KRR, there is a plethora of aims and applications in view when representing a knowledge of an agent, including fields beyond AI like software engineering, databases and robotics.

» Several logics have been studied for the latter purposes, including non-monotonic, epistemic, temporal, many-valued and fuzzy logics.

The Logics of Formal Inconsistency (LFIs)

Contradiction, (in)consistency, and triviality

LFIs are a family of paraconsistent logics designed to express the notion(s) of consistency and inconsitency within the object language by employing a connective "$\circ$" (or "$\bullet$"), in which $\circ\alpha$ means that "α is consistent" (and $\bullet\alpha$ means that "α is inconsistent").

Accordingly, the principle of explosion is not valid in general, although this law is not abolished but restricted to the so-called "consistent sentences", a feature captured by the following law, which is referred to as the "principle of Gentle Explosion" (PGE):

$$\alpha, \neg\alpha, \circ\alpha \vdash \beta, \text{ for every } \beta, \text{ but } \alpha, \neg\alpha \nvdash \beta \text{ for some } \beta \quad (1)$$

The hierarchy of LFIs

Definition (**mbC**)

The logic **mbC** is defined over the language $\mathcal{L}$ (generated by the connectives $\wedge, \vee, \rightarrow, \neg, \circ$) by means of a Hilbert system as follows:

Axioms:

(A1) $\alpha \rightarrow (\beta \rightarrow \alpha)$
(A2) $(\alpha \rightarrow \beta) \rightarrow ((\alpha \rightarrow (\beta \rightarrow \delta)) \rightarrow (\alpha \rightarrow \delta))$
(A3) $\alpha \rightarrow (\beta \rightarrow (\alpha \wedge \beta))$
(A4) $(\alpha \wedge \beta) \rightarrow \alpha$
(A5) $(\alpha \wedge \beta) \rightarrow \beta$
(A6) $\alpha \rightarrow (\alpha \vee \beta)$
(A7) $\beta \rightarrow (\alpha \vee \beta)$
(A8) $(\alpha \rightarrow \delta) \rightarrow ((\beta \rightarrow \delta) \rightarrow ((\alpha \vee \beta) \rightarrow \delta))$
(A9) $\alpha \vee (\alpha \rightarrow \beta)$
(A10) $\alpha \vee \neg\alpha$

(bc1) $\circ\alpha \rightarrow (\alpha \rightarrow (\neg\alpha \rightarrow \beta))$

Inference Rule:

(Modus Ponens (MP)) $\alpha, \alpha \rightarrow \beta \vdash \beta$

The hierarchy of LFIs: mbC

Semantic Characterization

mbC can be characterized in terms of valuations over $\{0, 1\}$ (also called *bivaluations*), but cannot be semantically characterized by finite matrices.

Definition (Valuations for **mbC**)

A function $v : \mathbb{L} \to \{0, 1\}$ is a *valuation for* **mbC** if it satisfies the following clauses:

(Biv1) $v(\alpha \wedge \beta) = 1 \iff v(\alpha) = 1$ and $v(\beta) = 1$

(Biv2) $v(\alpha \vee \beta) = 1 \iff v(\alpha) = 1$ or $v(\beta) = 1$

(Biv3) $v(\alpha \to \beta) = 1 \iff v(\alpha) = 0$ or $v(\beta) = 1$

(Biv4) $v(\neg\alpha) = 0 \implies v(\alpha) = 1$

(Biv5) $v(\circ\alpha) = 1 \implies v(\alpha) = 0$ or $v(\neg\alpha) = 0$.

The hierarchy of LFIs: mbC valuations are governed by quasi-matrices (distinct to truth-tables)

Consistency $\neq$ non-contradictoriness
Inconsistency $\neq$ contradictoriness

α	$\neg\alpha$	$\circ\alpha$	$\neg\circ\alpha$	$\alpha \wedge \neg\alpha$	$\neg(\alpha \wedge \neg\alpha)$	
1	1	0	1	1	1	v_1
					0	v_2
	0	1	1	0	1	v_3
			0	0	1	v_4
		0	1	0	1	v_5
0	1	1	1	0	1	v_6
			0	0	1	v_7
		0	1	0	1	v_8

The hierarchy of LFIs: mbC valuations are governed by quasi-matrices (distinct to truth-tables)

Consistency $\neq$ non-contradictoriness
Inconsistency $\neq$ contradictoriness

mbC separates the notions of consistency and non-contradictoriness:

$$\circ\alpha \vdash_{mbC} \neg(\neg\alpha \wedge \alpha) \text{ but the converse does not hold}$$

mbC also separates the notions of inconsistency (= non-consistency) and contradictoriness:

$$\alpha \wedge \neg\alpha \vdash_{mbC} \neg\circ\alpha \text{ but the converse does not hold}$$

The hierarchy of LFIs: Reassessing classical logic

Derivability Adjustment Theorem

» **LFIs** are at the same time subsystems and extensions of **CPL**.

» They can be seen as classical logic extended by two connectives: a paraconsistent negation and a consistency connective (or an inconsistency one, dual to it).

Theorem (Derivability Adjustment Theorem)

$X \vdash_{CPL} \alpha$ *if and only if* $\circ(Y), X \vdash_{mbc} \alpha$ *for some* $Y \subseteq \mathcal{L}_0$.

The hierarchy of LFIs

Remark (derived bottom particle and strong negation)

The *falsum* (or bottom particle) is defined in **mbC** by means of the formula $\bot_\beta =_{def} \beta \wedge \neg\beta \wedge \circ\beta$, for any formula β. From this, the classical (or strong) negation is defined in **mbC** by ${\sim_\beta}\alpha =_{def} (\alpha \to \bot_\beta)$. Since $\bot_\beta$ and $\bot_{\beta'}$ are interderivable in **mbC**, for any β and β', then ${\sim_\beta}\alpha$ and ${\sim_{\beta'}}\alpha$ are also interderivable. Hence, the strong negation of α will be denoted simply by ${\sim}\alpha$. The same applies to $\bot$.

The hierarchy of LFIs

Definition (Extensions of mbC)

Consider the following axioms:

(ciw) $\circ\alpha \vee (\alpha \wedge \neg\alpha)$

(ci) $\neg\circ\alpha \to (\alpha \wedge \neg\alpha)$

(cl) $\neg(\alpha \wedge \neg\alpha) \to \circ\alpha$

(cf) $\neg\neg\alpha \to \alpha$

(ce) $\alpha \to \neg\neg\alpha$

Some interesting extensions of **mbC** are the following:

mbCciw = **mbC**+(ciw)
mbCci = **mbC**+(ci)
bC = **mbC**+(cf)
Ci = **mbC**+(ci)+(cf) = **mbCci**+(cf)
mbCcl = **mbC**+(cl)
Cil = **mbC**+(ci)+(cf)+(cl) = **mbCci**+(cf)+(cl) = **mbCcl**+ (cf) + (ci) = **Ci**+(cl)
Cie = **Ci**+ (ce)

Some applications in KRR

Paraconsistent Belief Change

Reasoning with (in)consistency

» Paraconsistent logics allow the acceptance of contradictory beliefs, thus **admitting the definition of new operations**: including *external revision* (tolerating an intermediate temporary contradictory belief set) and *semi-revision* (a generalization of the former, in which a new piece of information does not have priority over the previously accepted ones) (Testa, Coniglio, Ribeiro 2017).

» Furthermore, the **axiomatic approach to (in)consistency allows new, more realistic epistemic attitudes**.

Paraconsistent Belief Change

Reasoning with (in)consistency: a toy example

A diamond was stolen, and only Adam and Bob were present. Since there are no proofs (but only evidences) against them, Hercule Poirot cannot consider them guilty. So his original belief set is $K_0 = \{\neg A, \neg B\}$ for "Adam is not guilty" and "Bob is not guilty".

Later on, the diamond was found in their car, so Poirot's belief set is expanded to $K_1 = \{\neg A, \neg B, A \vee B\}$.

Classically, $K_1 = \bot$, considering that $K = Cn(K)$.

Now, suppose that Poirot's logic is Cie.
Then $K_1 \nvdash A$ and $K_1 \nvdash B$

A new investigation discovers that only Adam used the car since the diamond was stolen. Then, Poirot can assume that the supposition about Bob's innocence is indeed consistent, arriving to a new belief set:
$K_2 = \{\neg A, \neg B, A \vee B, \circ\neg B\}$. But now

$$K_2 \vdash \circ\neg B, \neg B, A \vee B \vdash \circ\neg B, \neg B, A \vee \neg\neg B \vdash A$$

and so

$$K_2 \vdash A \wedge \neg A \vdash \neg\neg A \wedge \neg A \vdash \bullet\neg A$$

That is, Adam is now proven to be guilty, and the initial supposition about his innocence was proven to be inconsistent.

Paraconsistent Belief Change

Formal consistency as an epistemic attitude

Definition (Possible epistemic attitudes in AGM$\circ$ (a paraconsistent version of AGM))

Let K be a given belief set. Then, a sentence α is said to be:

Accepted if $\alpha \in K$.

Rejected if $\neg\alpha \in K$.

Under-determined if $\alpha \notin K$ and $\neg\alpha \notin K$.

Over-determined if $\alpha \in K$ and $\neg\alpha \in K$.

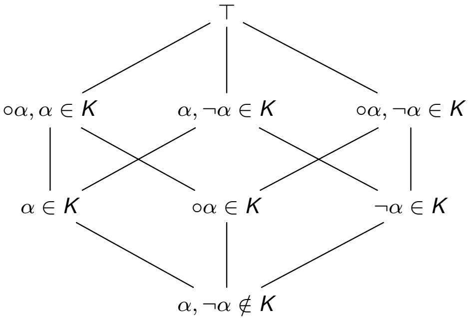
Consistent if $\circ\alpha \in K$.

Boldly accepted if $\circ\alpha \in K$ and $\alpha \in K$.

Boldly rejected if $\circ\alpha \in K$ and $\neg\alpha \in K$ (i.e. $\sim\alpha \in K$).

Paraconsistent Belief Change

Formal consistency as an epistemic attitude



Links from bottom to top mean proper inclusion.

Figure: Epistemic attitudes in AGM∘

Paraconsistent Belief Change

Formal consistency as an epistemic attitude

Example (Adapted from Hansson (1999))

i. Doris is not religious, but she has religious leanings. She does not believe that God exists ($G \notin K$), but it is possible for her to become a believer ($\sim G \notin K$).
  » since $\sim G \notin K$ then, if $G \in K$, $K \nvdash \bot$.

ii. Ellen, on the other hand, is a believer ($G \in K$). However, it may very well happen that she loses her faith so definitely that she can never become a believer in God again ($\circ\neg G \in K$).
  » since $\circ\neg G \in K$ then, if $\neg G \in K$, G will be strongly rejected.

iii. Florence is an inveterate doubter. Nothing can bring her to a state of firm (irreversible) belief ($\circ G \notin K$) and neither can she be brought to a state of firm disbelief ($\circ\neg G \notin K$).
  » in this sceptic scenario, G will never be strongly accepted nor rejected (in the cases when $G \in K$ or $\neg G \in K$).

Sound probabilistic reasoning under contradiction

» Paraconsistent probabilities can be regarded as degrees of belief that a rational agent attaches to events, even if such degrees of belief might be contradictory. Thus it is not impossible for an agent to believe in the proposition α and $\neg\alpha$ and to be rational, if this belief is justified by evidence (Bueno-Soler and Carnielli 2016).

Sound probabilistic reasoning under contradiction

Definition

A probability function for a language $\mathcal{L}$ of a logic **L**, or a **L**-probability function, is a function $P : \mathcal{L} \mapsto \mathbb{R}$ satisfying the following conditions, where $\vdash_L$ stands for the syntactic derivability relation of **L**:

1. Non-negativity: $0 \leq P(\varphi) \leq 1$ for all $\varphi \in \mathcal{L}$
2. Tautologicity: If $\vdash_L \varphi$, then $P(\varphi) = 1$
3. Anti-tautologicity: If $\varphi \vdash_L$, then $P(\varphi) = 0$
4. Comparison: If $\psi \vdash_L \varphi$, then $P(\psi) \leq P(\varphi)$
5. Finite additivity: $P(\varphi \vee \psi) = P(\varphi) + P(\psi) - P(\varphi \wedge \psi)$

This collection of meta-axioms, by assuming appropriate $\vdash_L$ (for instance, by taking the classical, intuitionistic or paraconsistent derivability relation) defines distinct probabilities, each one deserving a full investigation.

Sound probabilistic reasoning under contradiction

A toy example

The following example uses the system **Ci**, a member of the **LFI** family with some features that make it reasonably close to classical logic; it is appropriate, in this way, to define a generalized notion of probability strong enough to enjoy useful properties.[1]

Definition

Paraconsistent Bayes' Theorem

$$P(\alpha/\beta) = \frac{P(\beta/\alpha) \cdot P(\alpha)}{P(\beta/\alpha) \cdot P(\alpha) + P(\beta/\neg\alpha) \cdot P(\neg\alpha) - P(\beta/\bullet\alpha) \cdot P(\bullet\alpha)}$$

[1] Recall that $\bullet\alpha \leftrightarrow \alpha \wedge \neg\alpha$.

Sound probabilistic reasoning under contradiction

A toy example

A Bayesian spam filter analyses suspicious characteristics in the body of the messages:

- » Headers (senders and message paths)
- » Presence of HTML code
- » Colors
- » Suspicious word
- » Suspicious expressions, etc.

Sound probabilistic reasoning under contradiction:

A toy example

Some words (or characteristics) W are suspicious, while some are just dubious.

Suspicious

- “Sex”
- “Buy”
- “Cash”
- HTML codes
- Links, etc,

Dubious

- “Free”
- “Now”, etc.

Suppose that the historical probabilities coming from the training data are:

$P(S) = 30\%$ (the probability that any given email is spam),
$P(H) = P(\neg S) = 85\%$ (the probability that any given email is ham)

This information is measured by independent methods, and that is why $P(S) + P(\neg S) > 1$.

Suppose also that, for a given word or characteristic W:

$P(W/S)$ is the conditional probability that a message contains W given that it is spam, computed as the proportion of spam containing W

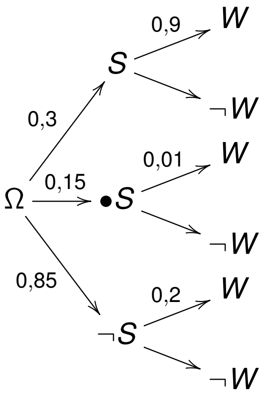
$P(W/H) = P(W/\neg S)$ is the conditional probability that a message contains W given that is ham, also computed as the proportion of ham documents containing W.

Of course, here $S \wedge H \neq \emptyset$, for $H = \neg S$ and $P(S \wedge \neg S) = P(\bullet S) = 0,15$.

As an example, suppose:

$P(W/S) = 0,9$
$P(W/\neg S) = 0,2$

What is the probability that an email is spam given that the email contains the suspicious W?



Plugging all the values in the paraconsistent version of Beyes' Theorem, we get $P(S/W) = 61,6\%$

Comparing paraconsistent and classical analysis

Forcing consistency maintenance

Towards NO SPAM	Half and half	Towards SPAM
$P(S) = 30\%$ $P(\neg S) = 70\%$ $P(W/S) = 90\%$ $P(W/\neg S) = 20\%$	$P(S) = 22,5\%$ $P(\neg S) = 77,5\%$ $P(W/S) = 90\%$ $P(W/\neg S) = 20\%$	$P(S) = 15\%$ $P(\neg S) = 85\%$ $P(W/S) = 90\%$ $P(W/\neg S) = 20\%$
Result	Result	Result
$P(S/W) = 65,9\%$	$P(S/W) = 56,6\%$	$P(S/W) = 44,3\%$

Paraconsistent probability finds a compromised solution!

Using paraconsistent probabilities we get $P(S/W) = 61,6\%$

Other applications and further work

» DLs can be expanded with paraconsistent, probabilistic and possibilistic tools, or with their combinations.

» Enhancing DLs with LFI-probabilities and possibility measures is a research in progress, and will represent a considerable step forward to DLs in regard to the representation of more realistic ontologies.

Other applications and further work

» A second problem concerns clarifying the concept of evidence. Rodrigues, Bueno-Soler, and Carnielli (2020) introduces the logic of evidence and truth – a Logic of Formal Inconsistency and Undeterminedness that extends Belnap–Dunn four-valued logic.

» This may represent an important leap forward into the clarification of the notion of evidence, each time more demanded in AI and KR.

Other applications and further work

» Paraconsistent Bayesian networks, notably when combined with paraconsistent belief revision and with belief maintenance systems can lead to a new approach to detecting and handling contradictions, and producing explanations for its conclusions.

» This is naturally relevant, for instance, in medical diagnosis, natural language understanding, forensic sciences and other areas where evidence interpretation is an important issue.

Paraconsisent Logics can expand our arsenal of reasoning tools

» Classical logic can be a burden.

» Paraconsistent logics can be added to our arsenal of rationality, especially when combined with quantitative methods : probability and evidence expanded.

» Our view goes against the traditional view that non-contradictoriness (i.e. classical consistency) is a requirement for rational belief.

» Contradictions can be informative! Other logic views on probability are, of course, possible if they regard data differently. So, why not to be a happy pluralist, and have more ways to compute your chances?

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